Session 6.2

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Notes to keep in mind

Make sure you have these things in your notes, because I will refer to them with the expectation that you have learned, memorized, or written them down.

1. Solving a system of equations with the **elimination method**

$$\begin{cases} 4x - 7y = -12 \\ -3x + 6y = 9 \end{cases} \xrightarrow{\text{multiply}} \begin{cases} 12x - 21y = -36 \\ -12x + 24y = 36 \end{cases} \xrightarrow{\text{add}} 3y = 0 \xrightarrow{\text{solve}} \boxed{y = 0} \xrightarrow{\text{plug in}} -3x+6(0) = 9 \xrightarrow{\text{solve}} \boxed{x = -3}$$

2. Factoring a polynomial from $x^2 + b * x + c$ into $(x + u)(x + v)$,
   (a) Remember that $b = u + v$ and $c = u * v$
   (b) Start by factoring out c, such as $24 = 1 * 24 = 2 * 12 = 3 * 8 = 4 * 6$
   (c) See if any pair of factors add up to equal b
   (d) If c is positive, that means u and v are both either positive or negative
   (e) If c is negative, one is positive and the other is negative

Main problems

1. Find the (x, y) solution to each of the following:

(a) $\begin{cases} 7x - 8y = -1 \\ y = 5x - 4 \end{cases}$ (c) $\begin{cases} -2x - 3y = -7 \\ y = 6x - 11 \end{cases}$ (e) $\begin{cases} 3x + 12y = -15 \\ x = 8y - 2 \end{cases}$

(b) $\begin{cases} -11x - 6y = 9 \\ y = -2x + 3 \end{cases}$ (d) $\begin{cases} -4x + 5y = -13 \\ y = -7x + 13 \end{cases}$ (f) $\begin{cases} -2x - 10y = -2 \\ x = 5y - 13 \end{cases}$

2. Find the (x, y) solution to each of the following:

(a) $\begin{cases} 3x + 5y = -35 \\ 6x + 6y = -54 \end{cases}$ (d) $\begin{cases} 4x + 4y = 4 \\ 6x + 2y = -2 \end{cases}$ (g) $\begin{cases} 4x + 3y = -7 \\ 3x + 5y = -19 \end{cases}$

(b) $\begin{cases} 6x + 4y = 6 \\ 2x + 4y = 2 \end{cases}$ (e) $\begin{cases} 2x + 4y = -14 \\ 5x + 3y = -21 \end{cases}$ (h) $\begin{cases} 5x + 6y = -37 \\ 3x + 5y = -25 \end{cases}$

(c) $\begin{cases} 3x + 6y = 21 \\ 4x + 2y = 4 \end{cases}$ (f) $\begin{cases} 6x + 2y = -6 \\ 4x + 3y = -9 \end{cases}$ (i) $\begin{cases} 6x + 2y = 10 \\ 4x + 3y = 5 \end{cases}$

3. For each of the following quadratic polynomials, **either** describe all of the transformations, **or** graph it and label five points. If you describe the transformations (how the graph differs from $y = x^2$), use phrases like, *"nothing"*, or *"up 2, then left 4, then reflected about x-axis"*.

(a) $y = x^2$
(b) $y = -x^2$
(c) $y = x^2 + 4$
(d) $y = x^2 - 3$
(e) $y = -x^2 - 2$
(f) $y = (x - 2)^2$
(g) $y = (x + 1)^2$
(h) $y = -(x + 3)^2$
(i) $y = 2x^2$
(j) $y = 1/2 * x^2$
(k) $y = (x + 5)^2 - 9$
(l) $y = (x - 4)^2 + 6$
(m) $y = -(x + 6)^2 + 10$
(n) $y = -(x - 3)^2 - 7$
(o) $y = -3(x - 7)^2$
(p) $y = 1/4 * (x - 1)^2 + 5$
(q) $y = -5(x + 4)^2 - 2$
(r) $y = (3x + 6)^2 + 1$

4. For each of the following transformations to $y = x^2$, write the quadratic equation in the form $y = c * (x + a)^2 + b$.

(a) Up 4
(b) Down 2
(c) Left 1
(d) Right 5
(e) Reflect about x-axis
(f) Up 2, then right 3
(g) Down 4, then left 5
(h) Reflect about x-axis, then right 1
(i) Reflect about x-axis, then up 4, then left 2
(j) Up 4, then reflect about x-axis
(k) Down 7, then reflect about x-axis, then right 3
(l) Up 4, then reflect about x-axis

5. Expand each of the following polynomials:

(a) $(x + 2)^2$
(b) $(x - 7)^2$
(c) $(x - 5)^2$
(d) $(x + 9)^2$
(e) $(x - 12)^2$
(f) $(x + 11)^2$
(g) $2(x + 3)^2$
(h) $3(x - 1)^2$

6. Factor each of the following:

(a) $y = x^2 + 6x + 9$
(b) $y = x^2 - 14x + 49$
(c) $y = x^2 - 18x + 81$
(d) $y = x^2 + 12x + 36$
(e) $y = x^2 + 24x + 144$
(f) $y = x^2 - 22x + 121$
(g) $y = 3x^2 - 30x + 75$
(h) $y = -4x^2 + 24x - 36$

7. Complete the squares of each graph, and describe the transformations happening in words:

(a) $x^2 - 6x + 14$
(b) $x^2 + 4x + 11$
(c) $x^2 + 2x + 10$
(d) $x^2 - 14 + 40$
(e) $x^2 - 12x + 12$
(f) $x^2 + 2x - 4$
(g) $x^2 - 6x - 6$
(h) $x^2 + 16x - 10$
(i) $x^2 + 24x + 100$
(j) $x^2 + 14x - 9$
(k) $x^2 - 18x + 53$
(l) $x^2 + 8x + 27$
(m) $x^2 + 22x - 21$
(n) $x^2 - 3x + 1$
(o) $-x^2 - 14x + 14$
(p) $-x^2 - 6x + 13$
(q) $4x^2 - 4x + 20$
(r) $2x^2 - 2x + 3$
(s) $-2x^2 + 28x - 7$
(t) $-2x^2 - 2x + 4$
(u) $-3x^2 - 24x + 24$

Counting and probability problems

1. Find the **probability** of drawing each type of card from a standard 52-card poker deck.
   (a) Draw an ace?
   (b) Draw a heart?
   (c) Draw a face card?
2. Suppose you're rolling two dice. How many ways can each event happen?
   (a) Rolling two 6's?
   (b) Rolling a 5 and a 4?
   (c) Rolling two evens?
   (d) Rolling a sum of 3?
   (e) Rolling a sum of 5?
   (f) What is the highest probability sum?
3. How many ways are there to sort each of the following in order?
   (a) Three students
   (b) Four different mugs
   (c) Ten college applications
4. Consider a class of eight students. How many ways can I order them in line with the following restrictions:
   (a) No restrictions?
   (b) Ederson must be in the front of the line?
   (c) Chris must be in the back so I can see where the line ends easily?
   (d) I have Mykal and Jordan stand with each other in line because I find it amusing?
   (e) Ederson, Max, and Enzo insist on standing with each other?
   (f) I need Christian and George to be separated?
5. Suppose I have a classroom of 10 kids playing jeopardy, and prizes are awarded to 1st, 2nd, and 3rd place. How many possible rankings are there for the top 3 where:
   (a) There are no restrictions?
   (b) Jose was playing, and he rage quit after one question?
   (c) Antonio and Brandon were colluding, so they are next to each other in the ranking? (Note: if one gets 3rd, then the other could get 4th and not be in the top 3!)
6. A substitution cipher is derived from orderings of the alphabet. How many ways can the 26 letters of the alphabet (21 consonants and 5 vowels) be ordered if each letter appears exactly once and:
   (a) there are no other restrictions?
   (b) all five vowels must be next to each other?
   (c) no two vowels can be next to each other?
7. Suppose you draw two cards in order from a 52-card deck. What is the probability you draw each of the following?
   (a) A 2 and a 7?
   (b) Pair of Ace's?
   (c) Pair of 10's

(d) Two hearts?

(e) Two spades in order?

(f) Any two numbers in order?

(g) Two cards of different suits?

(h) Two cards of different numbers?

(i) Any two numbers not in order?

8. 10 computers are brought in for servicing (and machines are serviced one at a time). Of the 10 computers, 3 are PCs, 4 are Macs, 2 are Linux machines, and 1 is an Amiga. Assume that all computers of the same type are indistinguishable (i.e., all the PCs are indistinguishable, all the Macs are the indistinguishable, etc.).

   (a) In how many distinguishable ways can the computers be ordered for servicing?

   (b) In how many distinguishable ways can the computers be ordered if the first 5 machines serviced must include all 4 Macs?

   (c) In how many distinguishable ways can the computers be ordered if 1 PC must be in the first three and 2 PCs must be in the last three computers serviced?

9. You are planning out what courses you want to take for the next two years. You have 22 courses to schedule over 6 quarters. All the classes are distinct, and order of classes within a quarter doesnt matter. How many different course plans are possible:

   (a) if there are no restrictions? (For example, you could put all 22 in one quarter if you wanted. This is not a recommended course planning strategy.)

   (b) if you can only take at most 4 courses in any quarter?

10. Suppose you have a board and you want to go from the Southwest corner to the Northeast corner. In this game, you can only move up and right. How many paths are there from start to finish in the following worlds.

    (a) 3×3

    (b) 4×5

    (c) 6×6

    (d) 10×10

    (e) 8×8 but there is a rock on (4,4) so you need to avoid that square