Session 5.1

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Notes to keep in mind

Make sure you have these things in your notes, because I will refer to them with the expectation that you have learned, memorized, or written them down.

1. Solving a system of equations with the **substitution method**

$$\begin{cases} 5x - 2y = 8 \\ y = x - 1 \end{cases} \xrightarrow{\text{substitute}} 5x - 2(x-1) = 8 \xrightarrow{\text{solve}} \boxed{x = 2} \xrightarrow{\text{plug in}} y = (2) - 1 \xrightarrow{\text{solve}} \boxed{y = 1}$$

2. Solving a system of equations with the **elimination method**

$$\begin{cases} 4x - 7y = -12 \\ -3x + 6y = 9 \end{cases} \xrightarrow{\text{multiply}} \begin{cases} 12x - 21y = -36 \\ -12x + 24y = 36 \end{cases} \xrightarrow{\text{add}} 3y = 0 \xrightarrow{\text{solve}} \boxed{y = 0} \xrightarrow{\text{plug in}} -3x + 6(0) = 9 \xrightarrow{\text{solve}} \boxed{x = -3}$$

3. Factoring a polynomial from $x^2 + b * x + c$ into $(x + u)(x + v)$,

    (a) Remember that $b = u + v$ and $c = u * v$

    (b) Start by factoring out c, such as $24 = 1 * 24 = 2 * 12 = 3 * 8 = 4 * 6$

    (c) See if any pair of factors add up to equal b

    (d) If c is positive, that means u and v are both either positive or negative

    (e) If c is negative, one is positive and the other is negative

Main problems

1. Solve the following system of equations for the (x, y) solution

    (a) $\begin{cases} 9x - 4y = 15 \\ y = 3x - 3 \end{cases}$

    (b) $\begin{cases} -3x + 2y = 15 \\ y = -x + 4 \end{cases}$

    (c) $\begin{cases} 7y - 5x = -10 \\ x = -\frac{7}{5}y + 2 \end{cases}$

    (d) $\begin{cases} 5y - 7x = 4 \\ x = \frac{6}{7}y + 5 \end{cases}$

    (e) $\begin{cases} -2x + 3y = -1 \\ 2x + 5y = 25 \end{cases}$

    (f) $\begin{cases} 2x + y = 12 \\ -3x + y = 2 \end{cases}$

    (g) $\begin{cases} 2x + y = 9 \\ 3x - y = 16 \end{cases}$

    (h) $\begin{cases} x - 2y = 9 \\ x + 3y = 16 \end{cases}$

    (i) $\begin{cases} 2x - y = 9 \\ 3x + 4y = -14 \end{cases}$

    (j) $\begin{cases} 4x - 3y = 25 \\ -3x + 8y = 10 \end{cases}$

    (k) $\begin{cases} 3x + 4y = 52 \\ 5x + y = 30 \end{cases}$

    (l) $\begin{cases} 12x - 3y = 6 \\ 4x - y = 2 \end{cases}$

2. Simplify each of the following polynomials

(a) Add $-11x^2 - 2x - 15$ to $3x - 5$ (d) Multiply/expand $(x-6)^2$

(b) Subtract $-10x^2 - 10x + 1$ from $-4x^2 - 15x + 7$ (e) Multiply/expand $(x-2)(x-4)$

(c) Subtract $-14x^2 + 6$ from $-x^2 - 4x + 9$ (f) Multiply/expand $(x-5)(x+6)$

3. Graph each of the following quadratic polynomials. Describe how the graph differs from $y = x^2$ using phrases like, *"nothing"*, or *"up 2, then left 4, then reflected about x-axis"*.

(a) $y = x^2$

(b) $y = x^2 + 2$

(c) $y = x^2 - 6$

(d) $y = -x^2$

(e) $y = (x-4)^2$

(f) $y = (x+2)^2$

(g) $y = -(x+3)^2$

(h) $y = 2(x+5)^2$

(i) $y = -(x-5)^2 - 7$

(j) $y = (x+3)^2 + 5$

(k) $y = 3x^2$

(l) $y = 1/2 * x^2$

(m) $y = 2(x+5)^2$

(n) $y = 2(x+5)^2$

4. For each of the following transformations to $y = x^2$, write the quadratic equation in some form.

(a) Up 3

(b) Down 7

(c) Right 2

(d) Left 5

(e) Left 3, then down 7

(f) Right 3, then up 4

(g) Left 2, then down 5

(h) Reflected about x-axis

(i) Left 3, then reflected about x-axis

(j) Down 4, then reflected about x-axis

(k) Left 13, then up 7, then reflected about x-axis

(l) Reflected about x-axis, then up 7, then left 13

5. Factor each of the following, and list the x-intercepts:

(a) $y = x^2 + 6x + 9$

(b) $y = x^2 + 24x + 144$

(c) $y = x^2 - 18x + 81$

(d) $y = x^2 - 10x + 25$

(e) $y = x^2 - 22x + 121$

(f) $y = 3x^2 - 12x + 12$

(g) $y = -2x^2 - 28x - 98$

(h) $y = x^2 - 49$

(i) $y = x^2 - 121$

(j) $y = x^2 - 16$

(k) $y = 3x^2 - 75$

(l) $y = 4x^2 - 9$

(m) $y = 16x^2 - 36$

(n) $y = x^2 - 144/9$

(o) $y = x^2 - 81/16$

(p) $y = x^2 + 10x + 21$

(q) $y = x^2 + 13x + 40$

(r) $y = x^2 + 14x + 48$

(s) $y = x^2 - 2x - 8$

(t) $y = x^2 - 14x + 45$

(u) $y = x^2 - 18x + 17$

(v) $y = x^2 - 3x - 28$

(w) $y = x^2 - 8x - 65$

(x) $y = 3x^2 + 9x - 30$

(y) $y = -2x^2 + 36x - 34$

(z) $y = -4x^2 + 12x + 216$

6. Complete the squares of each graph, and describe the shift happening in words.

(a) $x^2 + 4x + 20$

(b) $x^2 + 6x + 12$

(c) $x^2 - 10 + 30$

(d) $x^2 - 2x - 15$

(e) $x^2 + 6x - 5$

(f) $x^2 - 10x + 2$

(g) $x^2 - 14x + 20$

(h) $x^2 - 8x - 5$

(i) $x^2 + 16x + 30$

(j) $-x^2 + 4x + 3$

(k) $-x^2 - 8x + 24$

(l) $-x^2 - 6x + 7$

More problems

1. Work on 2010 ICTM 1A/2A: http://www.ilmathcontest.com/hs/Questions/Reg/R10A.pdf

2. Use the "Noah sheets"