Session 4.2

Mr. Hernandez: josehdz@cs.stanford.edu

Notes to keep in mind

Make sure you have these things in your notes, because I will refer to them with the expectation that you have learned, memorized, or written them down.

1. Solving a system of equations with the **substitution method**

$$\begin{cases} 5x - 2y = 8 \\ y = x - 1 \end{cases} \xrightarrow{\text{substitute}} 5x - 2(x-1) = 8 \xrightarrow{\text{solve}} \boxed{x = 2} \xrightarrow{\text{plug in}} y = (2) - 1 \xrightarrow{\text{solve}} \boxed{y = 1}$$

2. Solving a system of equations with the **elimination method**

$$\begin{cases} 4x - 7y = -12 \\ -3x + 6y = 9 \end{cases} \xrightarrow{\text{multiply}} \begin{cases} 12x - 21y = -36 \\ -12x + 24y = 36 \end{cases} \xrightarrow{\text{add}} 3y = 0 \xrightarrow{\text{solve}} \boxed{y = 0} \xrightarrow{\text{plug in}} -3x+6(0) = 9 \xrightarrow{\text{solve}} \boxed{x = -3}$$

3. Characteristics of a polynomial, such as $ax^2 + bx + c$, or, more generally $ax^n + bx^{n-1} + \ldots + z$
   (a) The **degree** of a polynomial is the highest variable exponent, such as 2 or n
   (b) The **leading coefficient** is the coefficient of the variable with the highest degree, such as a
   (c) The **constant term** is the number without a variable next to it, such as c or z

Main problems

1. Find all points on the following graphs with the specified value of y
   (a) $y = |\frac{3}{4}x + \frac{3}{8}|$ where $y = 3$
   (b) $y = |3 - \frac{2}{3}x|$ where $y = 5$
   (c) $y = -\frac{1}{2} * |\frac{2}{5}x + 4| - 5$ where $y = -9$

2. Find the equation for the line passing through both points, then a parallel one, and a perpendicular one:
   (a) $(-3, 7)$ and $(3, -1)$
   (b) $(-2, -5)$ and $(6, 1)$

3. For each of the systems of equations, find the (x, y) solution.

   (a) $\begin{cases} 4x - 7y = 14 \\ y = 3 \end{cases}$
   (b) $\begin{cases} -4x + 5y = -15 \\ y = -5 \end{cases}$
   (c) $\begin{cases} -5x - 12y = 10 \\ x = 8 \end{cases}$
   (d) $\begin{cases} 4x - 3y = 8 \\ y = x + 1 \end{cases}$
   (e) $\begin{cases} -2x - 5y = 14 \\ y = -x + 4 \end{cases}$
   (f) $\begin{cases} 12y + 6x = 11 \\ x = \frac{4}{3}y - 2 \end{cases}$
   (g) $\begin{cases} 7y - 9x = -18 \\ x = -\frac{1}{2}y + 3 \end{cases}$
   (h) $\begin{cases} 2x + y = 9 \\ 3x - y = 16 \end{cases}$
   (i) $\begin{cases} 2x - y = 9 \\ 3x + 4y = -14 \end{cases}$

(j) $\begin{cases} 12x - 3y = 6 \\ 4x - y = 2 \end{cases}$ (k) $\begin{cases} 3x - 5y = 23 \\ 5x + 7y = 0 \end{cases}$ (l) $\begin{cases} -x + 3y = 4 \\ -6y + 2x + 8 = 0 \end{cases}$

4. Simplify each of the following polynomials
   (a) Add $2x^2 - 4x + 1$ to $-6x^2 - 7x - 5$
   (b) Add $-8x^2 + 11x - 6$ to $-7x^2 - 9x + 14$
   (c) Subtract $-2x^2 - 3x + 11$ from $4x^2 + 7x - 3$
   (d) Subtract $5x^2 - 4x - 7$ from $6x^2 - 5x + 3$
   (e) Multiply/expand $(x + 3)^2$
   (f) Multiply/expand $(x + 3)(x + 7)$
   (g) Multiply/expand $(x - 2)(x + 5)$
5. Graph each of the following quadratic polynomials. Denote the min/max point and x-intercept(s).
   (a) $y = x^2$
   (b) $y = x^2 + 2$
   (c) $y = x^2 - 6$
   (d) $y = 3x^2$
   (e) $y = -x^2$
   (f) $y = 1/2 * x^2$
   (g) $y = (x - 4)^2$
   (h) $y = (x + 2)^2$
   (i) $y = -(x + 3)^2$
   (j) $y = 2(x + 5)^2$
   (k) $y = -(x - 5)^2 - 7$
   (l) $y = (x + 3)^2 + 5$
6. In general, what happens if we add c (a constant)?
7. In general, what happens if we multiply the polynomial by -1?
8. In general, what happens if we add c (a constant) inside the quadratic?
9. Factor each of the following, and list the x-intercepts:
   (a) $y = x^2 - 10x + 25$
   (b) $y = x^2 + 12x + 36$
   (c) $y = x^2 - 16x + 64$
   (d) $y = x^2 - 4x + 4$
   (e) $y = x^2 + 22x + 121$
   (f) $y = 2x^2 + 16x + 32$
   (g) $y = -4x^2 + 8x - 4$
   (h) $y = x^2 - 36$
   (i) $y = x^2 - 16$
   (j) $y = x^2 - 121$
   (k) $y = 3x^2 - 27$
   (l) $y = 9x^2 - 25$
   (m) $y = x^2 - 121/16$
   (n) $y = x^2 + 8x + 15$
   (o) $y = x^2 + 7x + 12$
   (p) $y = x^2 + 2x - 15$
   (q) $y = x^2 + 4x - 21$
   (r) $y = x^2 - 11x + 28$
   (s) $y = x^2 - 5x - 36$
   (t) $y = x^2 - 12x - 45$
   (u) $y = 3x^2 + 9x - 30$
   (v) $y = -2x^2 + 36x - 34$
   (w) $y = -4x^2 + 12x + 216$

More problems

1. Work on 2014 ICTM 1A/2A: http://www.ilmathcontest.com/hs/Questions/Reg/R14A.pdf
2. Use the "Noah sheets": http://teachers.edenpr.org/ mkingsbury/mathteam/NoahSheets.pdf